Příklady k přednášce 2
Exercises for Lecture 2

1. Calculate e^{At} for

$$A = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

using two different methods.

2. Consider the system $\dot{x} = Ax + Bu$, $y = Cx + Du$ with $B = 0$, $D = 0$ and

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}.$$

Select $x(0)$ in such a manner that $y(t) = te^{-t}$, $t \geq 0$.

3. Consider the system $x(k+1) = Ax(k) + Bu(k)$, with $x(0) = x_0$ and $k \geq 0$, where

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad x_0 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

Determine a sequence of inputs so that the state remains at x_0, i.e., so that $x(k) = x_0$ for all $k \geq 0$.

4. Consider the system $\dot{x} = Ax + Bu$, $y = Cx + Du$ with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}, \quad D = 0.$$

Find an equivalent representation for the system, given by $\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u$, $y = \tilde{C}\tilde{x} + \tilde{D}u$, in which $\tilde{A}$ is in the Jordan canonical form. Check that the transfer functions of the two representations are equal.

5. Determine a system $\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k)$, $y(k) = \bar{C}\bar{x}(k) + \bar{D}u(k)$ obtained by sampling the continuous-time system $\dot{x} = Ax + Bu$, $y = Cx + Du$ given by

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

with sampling period T and shift α.