Příklady k přednášce 5
Exercises for Lecture 5

1. Consider the state equation

$$x(k+1)=\begin{bmatrix}1 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}x(k),\quad y(k)=\begin{bmatrix}0 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}.$$

   Determine all states that are (a) unobservable and (b) unconstructible.

2. Suppose that for the system

$$x(k+1)=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}x(k),\quad y(k)=\begin{bmatrix}0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}x(k)$$

   it is known that

$$y(1)=y(2)=y(3)=\begin{bmatrix}1\\ 1\end{bmatrix}.$$

   Based on this information, what can be said about the initial condition $x(0)$?

3. Consider the system $\dot{x}=Ax,\ y=Cx,$ where

$$A=\begin{bmatrix}0 & -1 & 1\\ 1 & -2 & 1\\ 0 & 1 & -1\end{bmatrix},\quad C=\begin{bmatrix}0 & 1 & 0\end{bmatrix}.$$

   Which eigenvalues of the system are unobservable?

4. Consider the state equation $\dot{x}=Ax+Bu,\ y=Cx,$ where

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 3w^2 & 0 & 0 & 2w\\ 0 & 0 & 0 & 1\\ 0 & -2w & 0 & 0\end{bmatrix},\quad B=\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix},\quad C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix},$$

   which is obtained by linearizing the nonlinear equations of motion of an orbiting satellite about a steady-state solution. In this equation, w is a nonzero constant. In the state $x=[x_1\ x_2\ x_3\ x_4]^T$, x_1 is the differential radius, while x_3 is the differential angle. In the input vector $u=[u_1\ u_2]^T$, u_1 is the radial thrust and u_2 is the tangential thrust. The output vector $y=[y_1\ y_2]^T$ comprises the measurable state variables x_1 and x_3.

   (a) Is this system controllable from u? Is the system observable from y?
   (b) Can the system be controlled if the radial thruster fails? What if the tangential thruster fails?
   (c) Is the system observable from y_1 only? From y_2 only?

5. Consider the discrete-time system that is obtained by sampling the continuous-time system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

with a sampling period T and shift α. Determine the values of T and α that destroy observability.