Příklady k přednášce 10
Exercises for Lecture 10

1. Consider the observable pair

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

   Determine a full-order full-state asymptotic observer with eigenvalues at, say, –2. Use

   (a) the direct method for eigenvalue assignment and

   (b) the observable version of Ackermann's formula.

2. Consider the first-order system $\dot{x} = Ax + w$, $y = Cx + v$, where $A = \alpha$, $C = 1$ and v, w are two independent Gaussian random processes with mean zero and variance $Ev^2 = V = 1$, $Ew^2 = W = \omega^2$, with real α and $\omega \geq 0$.

   Suppose that the system started operation in the infinite past. Determine the optimal state estimator that minimizes the estimation error variance. What is the ratio of the magnitudes of the estimator and system eigenvalues?

3. Consider the system $x(k+1) = Ax(k)$, $y(k) = Cx(k)$, where

$$A = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

   is in observer form. Design a deadbeat observer, in which the estimation error will become zero in a minimum number of steps L. What is L in this case?

4. Design an observer for the oscillatory system $\dot{x}(t) = v(t)$, $\dot{v}(t) = -\omega^2 x(t)$, using measurements of the velocity v. Place both observer poles at $s = -5\omega$.

5. A servomotor that drives a load is described by the equation $\ddot{\theta} + \dot{\theta} = u$, where θ is the shaft position (output) and u is the applied voltage. Suppose that only θ is available for measurement and derive a state-space representation of the servomotor.

   (a) Design a full-order full-state observer for the servomotor with eigenvalues at, say, – 5.

   (b) Design a reduced-order observer of order 1 with eigenvalue at – 5.